OGAP

OGAP Additive Framework

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Depending upon the strength of additive reasoning, students may move up and down between additive, transitional, counting, and non-additive reasoning and strategies as they interact with new topics or new concepts.

Problem Contexts

Counting
Quantities
Patterns

Additive Situations
Add to
Take from
Put together/Take apart
Compare

Concepts/Properties
Properties and Relationships
Magnitude

Base 10/Place Value
Concepts
Strategies

Problem Structures

Types of Items
Contextual
Non-contextual

Complexity of Addends
Single digit
Multiple digit
Multiples of 10, 100, 1000

Representations
Number Paths
Number Lines
Base 10
Ten Frames
Dot images

Properties and Relationships
Relationship between addition and subtraction
Commutative property
Associative property
Identity
Relationships between models, equations and contexts
Compensation
Constant difference

Addends
Two addends
Three or more addends
Compositions of 10
Relationship between addends

Meanings for Subtraction
Difference
Removal
Distance
Missing addend

Number of steps
Single step
Multi-step

	Result Unknown	Change Unknown	Start Unknown
Add To	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? 2 + 3 = ?	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? 2 + ? = 5	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? ? + 3 = 5
Take From	Five apples were on the table. I ate two apples. How many apples are on the table now? 5 – 2 = ?	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? 5 – ? = 3	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? ? – 2 = 3

	Total Unknown	Both Addends Unknown	Addend Unknown
Put Together/ Take Apart	Three red apples and two green apples are on the table. How many apples are on the table? 3 + 2 = ?	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? 5 = 0 + 5 5 = 5 + 0 5 = 1 + 4 5 = 4 + 1 5 = 2 + 3 5 = 3 + 2	Five apples are on the table. Three are red and the rest are green. How many apples are green? 3 + ? = 5 5 – 3 = ?

	Difference Unknown	Bigger Unknown	Smaller Unknown
Compare	("How many more?" version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? ("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? 2 + ? = 5 5 – 2 = ?	(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with "fewer"): Lucy has three fewer apples than Julie. Lucy has two apples. How many apples does Julie have? 2 + 3 = ? 3 + 2 = ?	(Version with "more"): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? (Version with "fewer"): Lucy has three fewer apples than Julie. Julie has five apples. How many apples does Lucy have? 5 – 3 = ? ? + 3 = 5

Adapted from the CCSSM (2010) Table 1 Common addition and subtraction situations.

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The strategies students use move back and forth across the levels as they learn new concepts and/or interact with new problem structures and contexts.

Additive

Transitional (Tens)

Counting (Ones)

Subitizing, Unitizing, Commutativity, Number Composition, Place value understanding

Additive Strategies

Traditional US algorithm

$$\begin{array}{r} {}^{1} \\ 18 \\ +27 \\ \hline 45 \end{array}$$

> Transparent algorithms
> Decomposes by place value and adds
> $18 + 27 = ?$
>
> $18 = 10 + 8$ $18 + 20 = 38$
> $27 = \underline{20 + 7}$ $38 + 7 = 45$
> $30 + 15 = 45$

Partial Sums

$$\begin{array}{r} 18 \\ +27 \\ \hline 30 \\ +15 \\ \hline 45 \end{array}$$

Uses properties

Associative Property
$7+2+8=7+(2+8)$

Commutative Property
$6+7=7+6$

Flexible compensation

$$\begin{array}{r} 18 + 27 \\ 20 + 25 \\ \hline 45 \end{array}$$

Fact Fluency

Fact Recall
$6+7=13$

Derived Facts
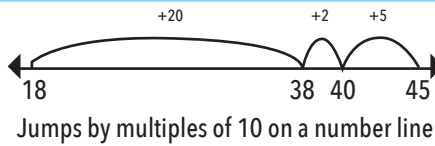$6+6=12$
$6+7=13$

Using 10s
$6+7$
$6+4=10$ and $10+3=13$

Transitional Strategies

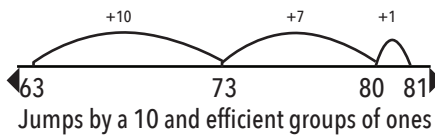**Efficient Use of a Model**

Jumps by multiples of 10 on a number line

Jumps by a 10 and efficient groups of ones

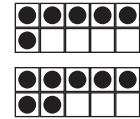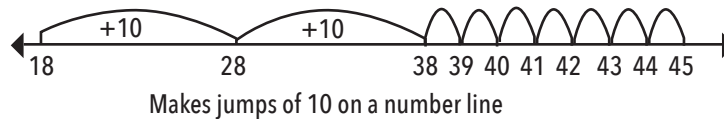**Inefficient decomposition to derive facts**

$6+7$
"I know $5+5$ is 10
and 3 more is 13"

Early Transitional Strategies

Adding inefficiently with or without a model

Adding on
by tens
$18+10=28$
$28+10=38$
$38+2=40$
$40+5=45$

Makes jumps of 10 on a number line

Combines or counts by 10s using base 10 representations
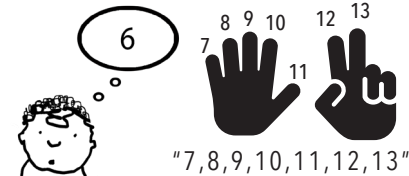

Unitizes on a model

Counting Strategies

Mental counting strategies:
- Count on from first
- Count on from larger

Counting on with physical or visual model: $6+7$

"7,8,9,10,11,12,13"

Early Counting Strategies

Direct modeling and counting from 1 with model (count 3 times)

"1,2,3,4,5,6,7,8,9,10,11,12,13"

Non-Additive Strategies
- Uses incorrect operation
- Models problem situation incorrectly
- Guesses
- Not enough information
- Uses procedures incorrectly

Underlying Issues/Errors
- Does not consider reasonableness of solution
- Error in counting, calculation, place value, property, equation, or model
- Units inconsistent or missing

OGAP

Additive Reasoning Progression – Subtraction

The strategies students use move back and forth across the levels as they learn new concepts and/or interact with new problem structures and contexts.

Additive

Additive Strategies

Traditional US algorithm

$$\begin{array}{r} {}^{6}\,{}^{1}\\ \cancel{7}\cancel{3}\\ -\,38\\ \hline 35 \end{array}$$

Transparent algorithms
Decomposes by place value and subtracts

$73 - 38 = ?$

$73 = 70 + 3$ $73 - 30 = 43$
$38 = 30 + 8$ $43 - 3 = 40$
$40 - 5 = 35$ $40 - 5 = 35$

Adding Up Efficiently
$38 + 2 = 40$
$40 + 33 = 73$
$2 + 33 = 35$

Constant Difference:
$73 - 38$
$+2 \quad +2$
$75 - 40 = 35$

Partial Difference:
73
$-\,38$
$40 - 5 = 35$

Fact Fluency
Fact Recall
Derived Facts

Inverse relationship between addition and subtraction
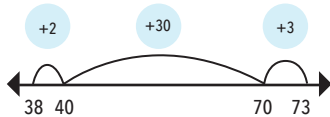"$7 - 2$ is 5 because $5 + 2$ is 7"

Using 10s: $13 - 8$
"$13 - 3 = 10$ and $10 - 5 = 15$"

Transitional (Place Value Parts)

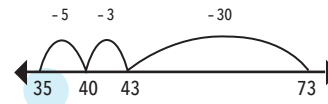## Transitional Strategies

Making efficient jumps by multiples of 10, 100, ...

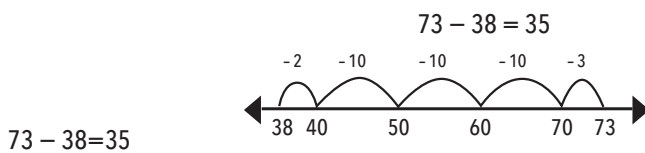$73 - 38 = 35$
Distance on number line

$73 - 38 = 35$
Jump back on number line

$13 - 8$

"$13 - 3$ is 10 and then 5 less is 5"

Unitizes on a model

Early Transitional Strategies

Subtracting inefficiently with or without a model

Using Base 10 representations

$73 - 38 = 35$
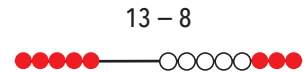
Jumps by tens on number line

$73 - 38 = 35$

$81 - 18 = 63$

Adding up by tens
$73 - 38 = ?$
$38 + 2 = 40$
$40 + 10 = 50$
$50 + 10 = 60$
$60 + 10 = 70$
$70 + 3 = 73$
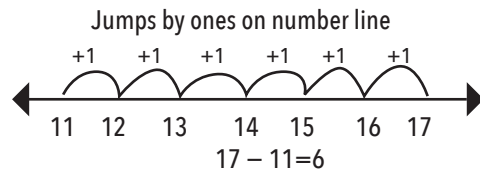$2 + 10 + 10 + 10 + 3 = 35$

Counting (Ones)

Counting Strategies

Counting back $(7 - 5)$
"7...6,5,4,3,2"

Counting up
"5...6, 7"

Counting back $(7 - 5)$
"7...6,5...that's 2"

Jumps by ones on number line

$17 - 11 = 6$

Early Counting Strategies

Direct modeling–separating from, separating to, or matching with model (counts 3 times)

$8 - 5 = 3$

Non-Additive Strategies
- Uses incorrect operation
- Models problem situation incorrectly
- Guesses
- Not enough information
- Uses procedures incorrectly

Underlying Issues/Errors
- Does not consider reasonableness of solution
- Error in counting, calculation, place value, property, equation, or model
- Units inconsistent or missing

Subitizing, Unitizing, Commutativity, Number Composition, Base 10 understanding

OGAP — Base Ten Number Progression

The strategies students use move back and forth across the levels as they learn new concepts and/or interact with new problem structures and contexts.

Subitizing, Mental Representations, Conservation, Base Ten Understanding

Base Ten

Application of Base Ten

- Flexible use of Base Ten understanding to solve problems

235
2 hundreds, 3 tens, 5 ones
23 tens & 5 ones
235 ones

- Multiplicative understanding of place value
  $235 = (2 \times 100) + (3 \times 10) + (5 \times 1)$
- Use of Base Ten for flexible and efficient computational strategies
- Extension of Base Ten understanding to decimal and negative numbers
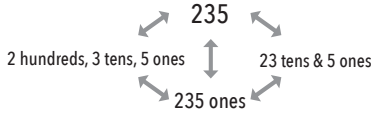
Number Composition by Place Value Parts

$200 + 30 + 5 = 235$
$200 = 20$ tens
$30 = 3$ tens
23 tens and 5 ones

Uses place value and number relationships to compare and consider relative magnitude

"63>59 because 6 tens is more than 5 tens"

Where does 66 go on the number line?
+16
0   50 60 66   100
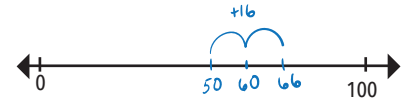
Transitional

Unitizing with Composite Units

100   200   230   235
10, 20, 21, 22, 23

Unitizes to locate and compare numbers
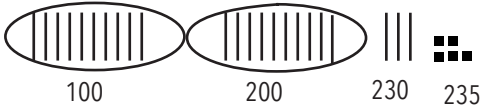
"63 is 4 more than 59"

Where does 66 go on the number line?
+10 +10 +10 +10 +10 +6
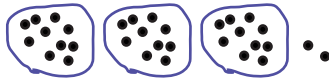0 10 20 30 40 50 60 66   100
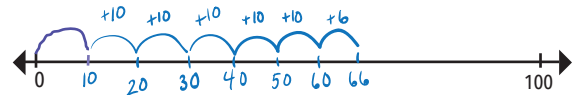
Early Unitizing (Sees quantities in groups when ones are visible)

Composes and decomposes quantities in groups
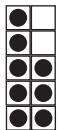
"5 and 3 is 8"
"2 less than 10 is 8"
"3 and 3 is 6 and 2 more is 8"

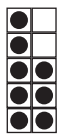"10, 20, 30, 31, 32"

"5 is 2 more than 3"
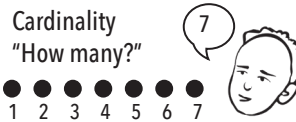
Where does 19 go on the number line?
0   5   10   15   20

Counting

Early Counting (Treats quantities as collections of ones)

"1,2,3,4,5,6,7,8"

Cardinality
"How many?"
7
1 2 3 4 5 6 7

Counts by ones to locate and compare numbers

One more/less than a number

1 2 3
4 5

1 2
3

"5 is more than 3"

Counts with one-to-one correspondence
"one" "two" "three" "four" "five" "six" "seven"

Disregards the unit
1, 2, 3, 4, 5

Where does 19 go on the number line?
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19
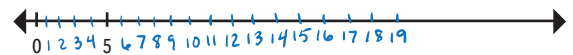
Precounting

Counts with some correspondence
"one" "two" "three" "four" "five..."

Rote counting

Perceptual comparing
"The second row is bigger"

Matching to compare

Not Counting or Comparing
- Guesses
- Not enough info
- Misinterprets problem situation or task

Underlying Issues/Errors
- Doesn't consider reasonableness of solution
- Sequence error
- Errors in units
- Numeral reversal
- Digit reversal
- Counting Error
- Model error
- Equation error