OGAP Multiplicative Framework

Updated July 2017

Depending upon the strength of multiplicative reasoning students may move up and down between multiplicative, transitional, additive, and non-multiplicative strategies as they interact with different problem situations and problem structures (Kouba & Franklin, 1995; VMP OGAP, 2006).

CONTEXTS

Equal Groups
Equal Measures
Measure Conversions
Multiplicative Comparison
Patterns
Unit Rate
Rectangular Area
Volume

CONCEPT/PROPERTY

Equations
Properties

Problem Structures

TYPES OF ITEMS
Context
Concept/property

LANGUAGE
Natural (e.g., every)
Mathematical (e.g., per)

MULTIPLICATIVE REPRESENTATIONS
Equal groups
Arrays
Area
Open area
Linear

COMPLEXITY OF NUMBERS
Single digit
Multiple digit
Powers of ten
Multiples of powers of ten
Fractions/decimals

NUMBER OF FACTORS
Two factors
More than two factors

TYPES OF DIVISION
Partitive
Quotative

UNDERSTANDING AND USE OF PROPERTIES
Associative
Commutative
Distributive
Equality
Identity
Multiplicative Inverse

UNDERSTANDING AND USE OF RELATIONSHIPS
Addition - Multiplication
Doubling and Halving
Model - Equation
Multiples and Factors
Meanings of Remainders
Context - Equation
Context - Model
Powers of ten

Problem Situations
Refers to known and unknown information in a problem. For example, in equal group problems there might be an unknown product, or an unknown number of equal groups, or an unknown quantity in a group. See page 4 for some examples. Also see the CCSSM page 89.

About OGAP Frameworks

OGAP Frameworks are based on mathematics education research on how students learn specific mathematics concepts, errors students make, and pre-conceptions or misconceptions that may interfere with learning new concepts or solving related problems. There are two major elements to an OGAP Framework that should be considered when analyzing student work or making instructional decisions: 1) Problem contexts and structures, and 2) OGAP Multiplication and Division Progressions.

This page identifies problem contexts and problem structures for multiplication and division problems. Pages 2 and 3, the OGAP Multiplication and Division Progressions, are tools to help teachers analyze evidence of student thinking and make instructional decisions. Page 4 has samples of different problem contexts.

For students to become strong multiplicative reasoners they must interact with a range of problem contexts, contextual situations, and problem structures. The CCSSM specifically identifies problem contexts at targeted grades on a progression from equal groups and measures, and area situations at grades 2 and 3 to measurement conversions, area, volume, multiplicative patterns, and multiplicative comparison problem situations at grades 4 and 5, and unit rates at grade 6. This progression, among other things, is designed to prepare students to engage in proportional situations using multiplicative reasoning.

Consistent with the CCSSM, the OGAP Progressions on pages 2 and 3 show the development of understanding and strategies from the link between repeated addition and multiplication in an equal groups model to the development of efficient and generalizable multiplicative strategies through the open area model, and understanding of place value, properties of operations, and relationships.
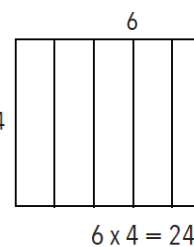
OGAP Multiplication Progression

OGAP

Multiplicative Strategies

Multiplicative

Algorithms

Partial Products

$$
\begin{array}{r}
16 \\
\times 42 \\
\hline
12 \\
20 \\
240 \\
400 \\
\hline
672
\end{array}
$$

Traditional

$$
\begin{array}{r}
{}^{21} \\
16 \\
\times 42 \\
\hline
32 \\
640 \\
\hline
672
\end{array}
$$

Distributive Property

$$
\begin{aligned}
4 \times 16 &= 4(10 + 6) \\
&= 4(10) + 4(6) \\
&= 40 + 24 \\
&= 64
\end{aligned}
$$

Associative Property

$$
\begin{aligned}
(8 \times 2) \times 5 &= 8(2 \times 5) \\
&= 8 \times 10 \\
&= 80
\end{aligned}
$$

Doubling & Halving

$$
\begin{aligned}
16 \times 4 &= 8 \times 8 \\
&= 64
\end{aligned}
$$

Known or Derived Fact

$$4 \times 6 = 24$$

Commutative Property

$$16 \times 4 = 4 \times 16$$

Powers of Ten

$$5 \times 400 = 5 \times 4 \times 10 \times 10$$

Transitional Strategies

Transitional

Open Area Model *Considers both dimensions of an array or area model*
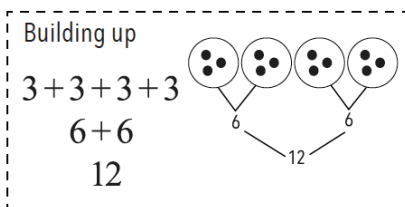

$$38 \times 26 = 988$$

$$15 \times 9 = 135$$
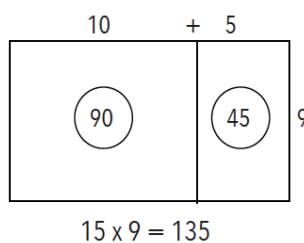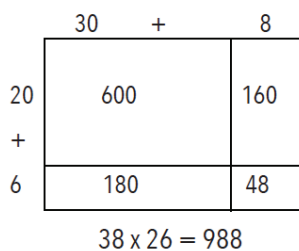
Area Model

Considers BOTH dimensions of an array or area model, moving away from needing to see every square unit
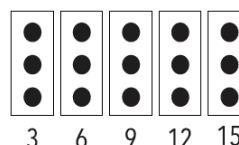
$$6 \times 4 = 24$$

Early Transitional Strategies

Skip Counting $3, 6, 9, 12, 15$

Building up

$$3 + 3 + 3 + 3$$
$$6 + 6$$
$$12$$

Skip Counting with a Model

3   6   9
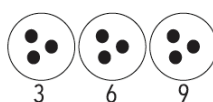
Equal groups in an array

3   6   9   12   15

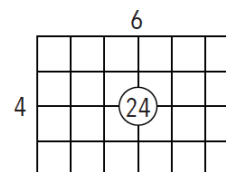*Considers only ONE dimension of an array or area model*
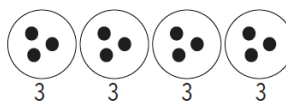
Area Model – $6 \times 4 = 24$
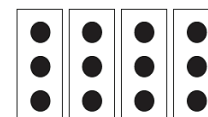

Considers BOTH dimensions of an array or area model

Additive Strategies

Additive

Repeated addition with or without a model – $3 \times 4 = 12$

$$3 + 3 + 3 + 3 = 12$$

3   3   3   3

Subitizing in small groups

$$3 + 3 + 3 + 3$$

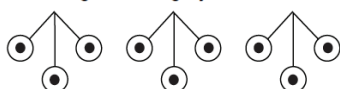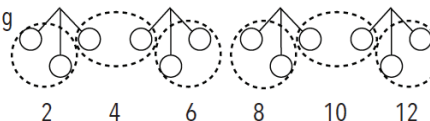### Early Additive Strategies

Modeling, counting by ones

Inconsistent Grouping

Modeling, counting by subgroups
$3 \times 4 = 12$

2   4   6   8   10   12

Non-Multiplicative Strategies

Adds or subtracts factors
Models factors incorrectly
Guesses

Uses incorrect operation
Not enough information
Uses procedures incorrectly

Underlying Issues and Errors

- Doesn't consider reasonableness of solution

- Error in: calculation, place value, vocabulary, property or relationship, equation, or model

Units inconsistent or missing

The strategies students use move back and forth across the levels as they learn new concepts and/or interact with new problems structures and contexts.

Unitizes into equal groups, uses visual models, understanding of place value, and properties of operations

OGAP Division Progression

OGAP

The strategies students use move back and forth across the levels as they learn new concepts and/or interact with new problems structures and contexts.

Unitizes into equal groups. uses visual models, understanding of place value, and properties of operations

Multiplicative

Multiplicative Strategies

Algorithms

Partial Quotients

```
17 | 585
    340 | x 20
    245
    170 | x 10
     75
     68 | x 4
      7 | 34 r 7
```

```
        4
       10
       20  34 r 7
17 | 585
    340
    245
    170
     75
     68
      7
```

Traditional

```
       34 r 7
17 | 585
    51↓
    75
    68
     7
```

Distributive Property

$35 \div 7 = (21 + 14) \div 7 = 3 + 2 = 5$

Treats the remainder appropriately given problem situation

Inverse relationship between multiplication and division

$35 \div 7 = 5 \qquad 7 \times ? = 35$

Known or Derived Fact

$21 \div 7 = 3$

Transitional

Transitional Strategies

Inefficient partial quotients

```
61 | 756      12 R 24
    183 | x 3
    573
    183 | x 3
    390
    183 | x 3
    207
    183 | x 3
     24
```

Trial and error to find a quotient

```
61 | 756
    732
     24      12 R 24
```

$2 \times 61 = 122$
$20 \times 61 = 1220$
$4 \times 61 = 244$
$8 \times 61 = 488$
$15 \times 61 = 915$
$10 \times 61 = 610$
$12 \times 61 = 732$

Early Transitional Strategies

Models in an array to find missing dimensions

$15 \div 3 = ? \qquad 3 \times ? = 15$

Skip Counts to find the number of "skips" with and without a model

$15 \div 3 = 5 \qquad 3, 6, 9, 12, 15 \text{ (5 skips)}$

```
0   3   6   9   12   15
```

Additive

Additive Strategies

Repeated subtraction or adding up to

```
61 | 350   5 R 45      61
    61               +61
    289              122
    61               +61
    228              183
    61               +61
    167              244
    61               +61
    106       350    305
    61       -305    +61
    45       r 45    366
```

Sharing out in equal groups as repeated subtraction or addition

There are 8 cookies to share equally with 4 children. How many cookies does each child get?

8 cookies ÷ 4 children = 2 cookies per child

Represents the 4 children with circles and then fills them equally

Twenty-four cookies were put into bags of 4 cookies each. How many bags were filled?

24 cookies ÷ 4 cookies per bag = 6 bags

Pulls out 4 cookies at a time until 24 cookies are used

Early Additive Strategies

Sharing out by ones

Sharing out randomly by subsets

Sharing 4, then 2, then 5, and so on

Non-Multiplicative Strategies

Adds or subtracts dividends/divisors

Models problem incorrectly

Guesses

Uses procedures incorrectly

Uses incorrect operation

Not enough information

Underlying Issues and Errors

- Doesn't consider reasonableness of solution

- Error in: calculation, place value, vocabulary, property or relationship, equation, or model

- Misinterprets the remainders

- Units inconsistent or missing

This is a derivative product of the Vermont Mathematics Partnership Ongoing Assessment Project (OGAP) which was funded by NSF (EHR-0227057) and the US DOE (S366A020002). © 2017. *OGAPMath LLC*

OGAP Sample Problem Contexts

Important: The examples below do not represent the full range of each problem context. * problem situations

Multiplication Examples	Division Examples
Equal Groups * *Unknown product example* Mark bought 12 boxes of crayons. Each box contained 8 crayons. How many crayons were there altogether? 12 boxes × 8 crayons = n crayons	* Unknown number in each group example* Mark had a box of 64 crayons. He shared the crayons equally with 4 people. How many crayons did each person get? 64 crayons ÷ 4 people = *n* crayons per box
Equal Measures * *Unknown product example* It takes 14 inches of ribbon to make one bow. How many inches of ribbon will it take to make 7 bows? 7 bows × 14 inches per bow = n inches	* *Unknown number of groups example* Sam has 15 yards of material. He is making a design that needs 3 yards per design. How many designs can Sam make? 15 yards ÷ 3 yards per design = *n* designs
Unit Rates * *Unknown product example* Sam works at the grocery store. He is paid $7.00 per hour. He worked 22 hours last week. How much money did Sam earn last week? 22 hours × $7.00/hour = *n* dollars	* *Unknown rate example* Sam earned $154.00 last week. He worked 22 hours. How much did Sam earn per hour? $154.00 ÷ 22 hours = *n* $/hour
Multiplicative Comparison * *Larger unknown example* The students in Mrs. Gilbert's class planted bean and corn seeds. The bean plants grow 3 times faster than the corn plants. When the corn plants measure 2 inches, how tall will the bean plants be? 2 inches × 3 = *n* height of bean plants	* *Multiplier unknown example* Bill's garden is 240 square feet. Leslie's garden is 20 square feet. How many times greater is Bill's garden than Leslie's garden? 240 square feet ÷ 20 square feet = *n* times bigger
Patterns **Larger unknown example* Tammy is decorating tables with vases of flowers for a party. She used the chart to the right to keep track of how many flowers she needed. Based on the information in the table, how many flowers does she need to fill 15 vases? 15 vases of flowers × 4 = *n* flowers	<table><tr><th>Number of Vases of Flowers</th><th>Number of Flowers</th></tr><tr><td>1</td><td>4</td></tr><tr><td>2</td><td>8</td></tr><tr><td>3</td><td>12</td></tr><tr><td>4</td><td>16</td></tr><tr><td>15</td><td></td></tr></table>
Measurement Conversion * *Unknown product example* Tammy is 5 feet tall. How many inches tall is Tammy? 5 feet × 12 inches/foot = *n* inches	* *Unknown quotient example* Tammy is 60 inches tall. How many feet tall is Tammy? 60 inches ÷ 12 inches/foot = *n* feet
Area * *Unknown product example* Linda's kitchen floor measures 12 feet by 7 feet. What is the area of Linda's kitchen floor? 12 feet × 7 feet = *n* square feet	* *Unknown factor example* Linda's kitchen floor is 150 square feet. The length of one dimension is 10 feet. What is the length of the other dimension of the kitchen floor? 150 square feet ÷ 10 feet = *n* feet
Volume * *Unknown product example* The dimensions of a fish tank are 10 inches by 12 inches by 18 inches. If you filled the tank to the top, how much water would be in the tank? (10 inches × 12 inches × 18 inches = *n* cubic inches)	* *Unknown factor example* A rectangular container holds 250 cubic inches of liquid. the base of the container is 10 inches and the depth 5 inches. What is the height of the container? 250 cubic inches ÷ (10 inches × 5 inches) = *n* inches
OGAP Equation Example: Write a story problem that goes with the equation 6 × 5 = 30.	**OGAP Property Example**: Ann knows the answer to 9 × 5. Explain how can she use this information to solve 45 ÷ 9?